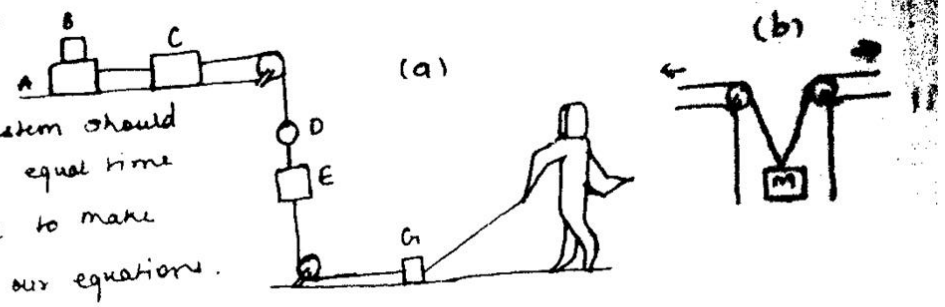
# WORKING WITH NEWTONS LAWS OF MOTION

## Steps you should follow when working on problems of newton's laws of motion

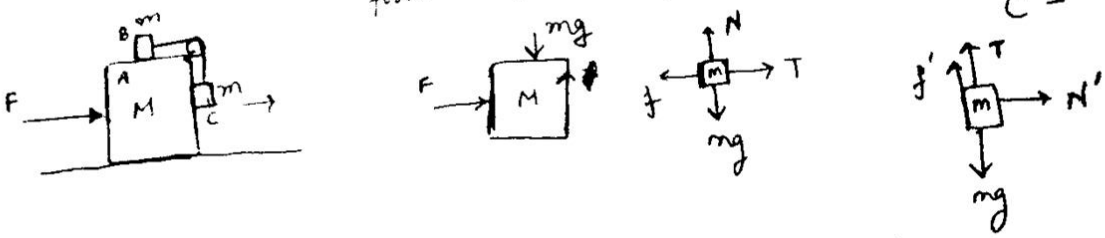
① **Decide The System** -




- Bodies taken in a system should cover equal distance in equal time. if not, then we have to make appropriate changes in our equations.

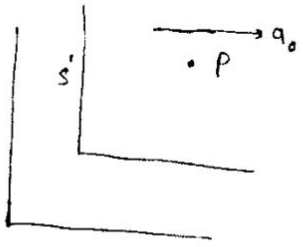
Ex - in (a) all bodies move equal distance in equal time but in (b) the distance rope is pulled is not the same as the distance covered by mass M.

② **Identify the forces and make FBD (free body diagram)** -




③ **Choose Axes and write equation** -

★ Concept of Pseudo force -




let S be an inertial frame of reference (frame at rest or moving with const. velocity)

let S' be the frame moving with some acceleration ($a_0$) with respect to S

Suppose a pain P moving with some acceleration then

$\vec{a}_{PS'} = \vec{a}_{PS} - \vec{a}_{S'S}$

$\vec{a}_{PS'}$ = acc$^r$ of P w.r.t S'

$\vec{a}_{PS}$ = acc$^r$ of P w.r.t S

$\vec{a}_{S'S}$ = acc$^r$ of S' w.r.t S

$\vec{a} = \vec{a}_{PS} - a_0$

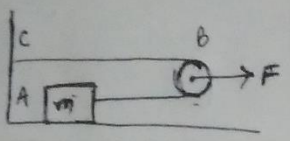
$m\vec{a} = m\vec{a}_{PS} - ma_0$

$\vec{a} = \dfrac{\vec{F} - m\vec{a}_0}{m}$

Here $-ma_0$ is the pseudo force that has to be subtracted in case we take S' as our frame of reference.

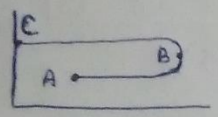
Now let us work out some problems to clear our concept -

(1)

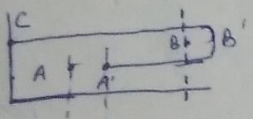



In this system if pulley is pulled by a force F and the surface is smooth then find acc$^r$ of A. (pulley is massless)

We observe the string here -




afterwards




Now we know

string length before = string length after

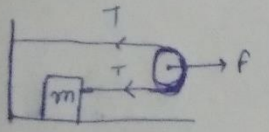
$AB + BC = A'B + BB' + BB' + BC$

$AB = A'B + 2BB'$

$AB - A'B = 2BB'$

$AA' = 2BB'$ $\left[\dfrac{d^2AA'}{dt^2} = 2\dfrac{d^2BB'}{dt^2}\right]$

let




$F - 2T = m_{pulley}\, a_{pulley}$ $[a_{mass} = 2 \times a_{pulley}]$

$F - 2T = 0$

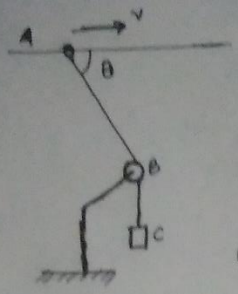
$T = \dfrac{F}{2}$

$a_{pulley} = \dfrac{T}{m} = \dfrac{F}{2m}$ $a_{mass} = a_{pulley}$.

$a_{mass} = \dfrac{T}{m} = \dfrac{F}{2m}$ $a_{pulley} = \dfrac{2}{2} \times a_{mass}$

$a_{pulley} = \dfrac{1}{2} \times \dfrac{F}{2m} = \dfrac{F}{4m}$

(a)

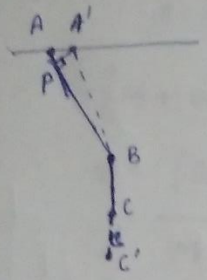



a smooth ring of mass m can slide on a fixed horizontal rod. A string tied to the ring passes over a fixed pulley B and carries a block C of mass M (= 2m). string makes an angle θ with the rod.

(i) show that if ring slides with velocity v, the block descends with velocity $v\cos\theta$.

(ii) find acc$^n$ initially if $\theta = 30^\circ$.

Observe the string for a small time $\Delta t$ -




$AB + BC = A'B + BC + CC'$

$AB = A'B + CC'$

$A'B = PB = AB - AP$

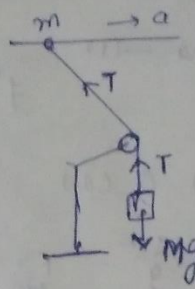
$\Rightarrow AB - A'B = AP$

$AP = AA'\cos\theta$

$\Rightarrow CC' = AA'\cos\theta$

$\frac{CC'}{\Delta t} = v = \frac{AA'\cos\theta}{\Delta t} = v\cos\theta$

Acc$^n$ —




Let acc$^n$ of ring be a

then

$Mg - T = Ma\cos\theta$

$T\cos\theta = ma$

$Mg = T + Ma\cos\theta$

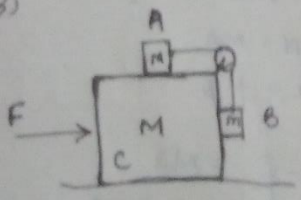
$= \frac{ma}{\cos\theta} + Ma\cos\theta$

$a = \frac{Mg\cos\theta}{m + M\cos^2\theta}$

at $\theta = 30^\circ$, $g = 9.8$ & $M = 2m$

we get $a = 6.78\ m/s^2$

(b)



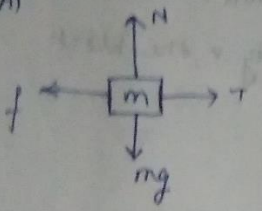

The horizontal surface below the bigger block is smooth. The co-efficient of friction between blocks is μ. Find the minimum and the maximum force F that can be applied in order to keep smaller block at rest w.r.t bigger block.

Solution - if we apply a force F the acc' of the system is

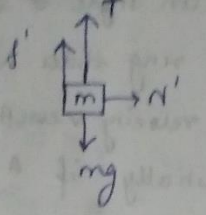
$$a = \frac{F}{M+2m}$$

Now, FBD - (When we apply minimum force)

(A)




(B)




$N = mg$ $\quad T - f = ma$

$T - \mu mg = ma$ —(1)

$N' = ma$

$T + f' = mg$

$f' = \mu N'$

$\Rightarrow T + \mu ma = mg$ —(2)

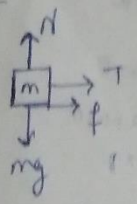
from (1) & (2)

$$a_{min} = \frac{1-\mu}{1+\mu} g \qquad F_{min} = (M+2m)\left(\frac{1-\mu}{1+\mu}\right) g$$

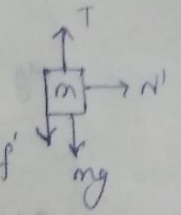
- When we apply a large force the block A slips towards left and B upwards so direction of friction changes -

FBD -

(A)

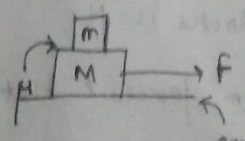



(B)

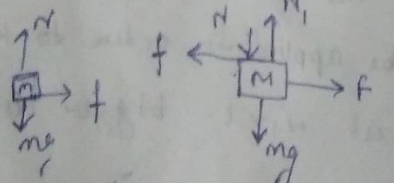



Now by solving we get $a_{max} = \left(\frac{1+\mu}{1-\mu}\right) g$ $\qquad f_{max} = (M+2m)\left(\frac{1+\mu}{1-\mu}\right) g$

(4) What maximum force F can be applied to the block of mass M so that blocks move together.




FBD -




$f = ma$

$\mu N = ma$

$a = \mu g$

$F - f = Ma$

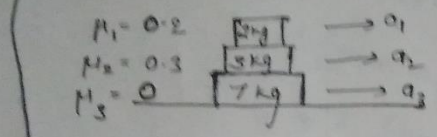
$F = Ma + \mu mg$

$F = \mu g (M + m)$

$\Rightarrow F \leq \mu g (M+m)$

(∵ We know at large force m slides backwards)

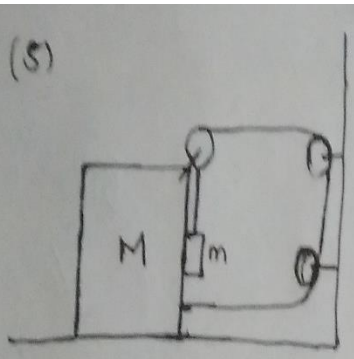
Do Yourself: Find the acc$^{n}$ $a_1, a_2, a_3$ of the three block if a horizontal force of 10N is applied on (a) 2kg block (b) 3kg block, (c) 7kg block




Answers:

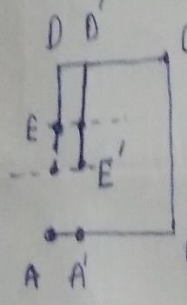
| | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| (a) | $3\ m/s^2$ | $0.4\ m/s^2$ | $0.4\ m/s^2$ |
| (b) | $5/6\ m/s^2$ | $5/6\ m/s^2$ | $5/6\ m/s^2$ |
| (c) | $5/6\ m/s^2$ | $5/6\ m/s^2$ | $5/6\ m/s^2$ |

(5)




Find the acc$^{n}$ of the block of Mass M. The co-efficient of friction between two blocks is $\mu_1$ and the bigger block and the surface is $\mu_2$.

Observe string




$AB + BC + CD + DE = A'B + BC + CD' + D'E + EE'$

$AB + CD + DE = A'B + CD' + D'E + EE'$

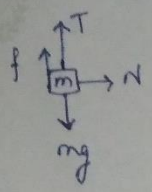
$(AB - A'B) + (CD - CD') = EE'$ $\quad [DE = D'E]$

$AA' + DD' = EE'$ $\quad [AA' = DD']$

$2AA' = EE'$

Let acc$^{n}$ of M be a then that of block m will be 2a.

FBD –

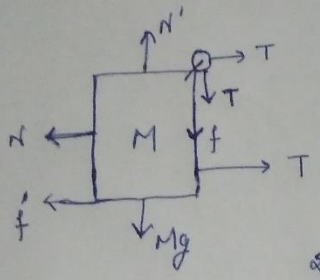



$mg - T - f = m(2a)$

$T = mg - \mu_1 ma - 2ma$ —— (1)

$N$ = mass of block (m) × acc$^{n}$ in horizontal direction

$= ma$




$N' = Mg + f + T$

$= Mg + mg - 2ma$

$2T - f' - N = Ma$

$2(mg - \mu_1 ma - 2ma) - \mu_2(Mg + mg - 2ma) - ma = Ma$

$a\left[M + m\{5 + 2(\mu_1 - \mu_2)\}\right] = [2m - \mu_2(M+m)]g$

$$a = \frac{[2m - \mu_2(M+m)]g}{M + m[5 + 2(\mu_1 - \mu_2)]}$$